DELHI PUBLIC SCHOOL BAHADURGARH

Sample Paper

CLASS XII MATHEMATICS (SET – 1)

M.M -100 **DURATION: 3 Hrs.**

DATE :

GENERAL INSTRUCTIONS:

1) All the questions are compulsory.
2) The question paper consists of 29 questions divided in three sections. Section A carries 10 questions of 1 mark each; Section B carries 12 questions of 4 marks each and section C carries 7 questions of 6 marks each.
3) All questions in section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4) Use of calculaters, log tables etc. is not permitted.

SECTION – A

Q.1) A matrix P of order 3x3 is such that $|Adj\ P|$ =64. Find the value of $|P|$.

Q.2) Find the angle between the vectors $\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $3\hat{\imath} - 2\hat{\jmath} + \hat{k}$.

Q.3) Write the position vector of a point which divides the line segment joining points with position vectors $2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$ and $-\hat{\imath} + \hat{\jmath} + \hat{k}$ externally in the ratio 1:4.

Q.4) Write the direction cosines of a line equally inclined to the three co-ordinate axes.

Q.5) Evalute $\tan^{-1}\{ 2 \cos (2 \sin^{-1} ½)\}$.

Q.6) Evaluate $\int \frac{1}{sin^2x.cos^2x}$ dx.

Q.7) $\int_{-1}^{1} |x|\ dx.$

Q.8)) If A=$\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$, find adj(A).

Q.9) For what value of x , the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular?

Q.10) If f(x) = x+ 7 and g(x) = x-7, find (fog) (7).

SECTION-B

Q.11) Let * be a binary operation on N x N defined by (a,b) * (c,d) = (a + c, b + d). Show that * is commutative and associative, Also , find the identity element for * on N x N if any.

Q.12) Show that $\cot^{-1}(7) + \cot^{-1}(8) + \cot^{-1}(18) = \cot^{-1}(3)$ **OR**

Show that $\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3} = \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$.

Q.13) If $y = (\sin x)^x + (\cos x)^{\tan x}$, find dy/dx.

Q.14) For what value of k, f(x) = $\begin{cases} 3x^2 - kx + 5, & 0 \le x < 2 \\ 1 - 3x, & 2 \le x \le 3 \end{cases}$ is continuous?

Q.15) Using properties of determinants , prove the following :

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3.$$

Q.16) If $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$, and $\vec{b}$ = i + 2j – 3k and c = 3i +4j – k then find a·(b x c) and (a x b)·c. Is a·(b x c) = (a x b) ·c? **OR**

If i + j + k, 2i + 5j, 3i + 2j - 3k and i – 6j –k are the position vectors of the points A, B, C, and D, find the angle between AB and CD. Deduce that AB And CD are collinear.

Q.17) Find the distance between the point P(6,5,9) and the plane determined by the points A(3,-1,2), B(5,2,4) and C (-1,-1,6). **OR**

Find the foot of the perpendicular drawn from the point P(2,4,-1) to the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$. Also find the equation of the perpendicular from P to the line.

Q.18) Prove that the curves $x = y^2$ and $xy = k$ intersect at right angles if $8k^2 = 1$.

Q.19) In a group of 100 families, 30 families like male child, 25 families like female child and 45 families feel both children are equal. If two families are selected at random out of 100 families, find the probability distribution of the number of families that feel both children are equal. What is the importance in the society to develop the feeling that both children are equal

Q.20) Solve the following differential equation: $x\frac{dy}{dx} + y = x \log x$; $x \neq 0$.

Q.21) Solve the following differential equation: $x^2\,dy + (xy + y^2)\,dx = 0$, given y = 1 at x = 1.

Q.22) Evaluate $\int_0^{\pi} \frac{x\tan x}{\sec x + \tan x}$ dx. **OR** Evaluate $\int_0^{\pi} \log(1 + \cos x)\, dx$

SECTION-C

Q.23) Show that the right circular cone of least curved surface area and given volume has an altitude equal to √2 times the radius of the base **OR**

Prove that the semi-vertical angle of the right circular cone of given total surface area and maximum volume is $\sin^{-1}(1/3)$.

Q.24) Find the area of that part of the circle $x^2 + y^2 = 16$ which is interior to the parabola $y^2 = 6x$.

Q.25) A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a 6. Why should we prefer truth to lie?

Q.26) Find the shortest distance between the lines :

$$\frac{x-3}{8} = \frac{y+19}{-16} = \frac{z-10}{7} \text{ and } \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

Also find the equation of the line of the shortest distance.

Q.27) Evaluate $\int \frac{\sqrt{x^2+1}\,\{\log(x^2-1) - 2\log x\}}{x^4}\, dx.$

OR

Evaluate $\int \frac{(x^2+1)\,(x^2+2)}{(x^2+3)\,(x^2+4)}$ dx.

Q.28) Find the inverse of the matrix A = $\begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$ by using elementary row transformations.

Q.29) An aeroplane can carry a maximum of 200 passengers. A profit of rs.400 is made on each first class ticket and a profit of rs. 300 is made on each second classs ticket. The airline reserves atleast 20 seats for first class. However , atleast four times as many passengers prefer to travel by second class than by first class. Detrmine how many tickets of each kind must be sold to maximize the profit. Form an LPP and solve it graphically.Why shouldn't there be only first class seats in aeroplanes?

DELHI PUBLIC SCHOOL BAHADURGARH

Sample Paper

CLASS XII MATHEMATICS (SET – 2)

M.M -100 **DURATION: 3 Hrs.**

DATE :

GENERAL INSTRUCTIONS:

1) All the questions are compulsory.
2) The question paper consists of 29 questions divided in three sections. Section A carries 10 questions of 1 mark each; Section B carries 12 questions of 4 marks each and section C carries 7 questions of 6 marks each.
3) All questions in section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4) Use of calculaters, log tables etc. is not permitted.

SECTION – A

Q.1) A matrix A of order 3x3 is such that $|3A|$ =K$|A|$. Find the value of K..

Q.2) Find the angle between the vectors $\vec{a}$ and $\vec{b}\ if\ |\vec{a}\,| = \sqrt{3}\,, |\vec{b}\,| = 2\ and\ \vec{a}.\vec{b} = 3$.

Q.3) If $\vec{a} = \hat{\imath} + 2\hat{\jmath} - \hat{k}\ and\ \vec{b} = 3\hat{\imath} + \hat{\jmath} - 5\hat{k}$, find a unit vector in direction of $\vec{a} - \vec{b}$.

Q.4) Write the direction cosines of a vector $\overrightarrow{PQ}$ joining the points P(1,5,4) and Q(4,1,-2).

Q.5) Evalute $\cos^{-1}\{-\sqrt{3}/2\}$.

Q.6) Evaluate $\int \frac{\sin x}{16 - 9cos^2 x}$ dx.

Q.7) $\int_0^3 |x-2|\ dx.$

Q.8)) If A = $\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$, find A^{-1}.

Q.9) For what value of x , the matrix $\begin{bmatrix} 4 & x+1 \\ 2 & 4 \end{bmatrix}$ is symmetric.

Q.10) If f(x) = x+ 7 and g(x) = x-7, find (gof) (2).

SECTION-B

Q.11) Consider f: $R_+ \rightarrow [4, \infty)\ given\ by\ f(x) = x^2 + 4$. Show that f is invertible with the inverse f^{-1} of f given by $f^{-1}(y) = \sqrt{y-4}$, where R_+ is the set of all non-negative real numbers.

Q.12) Show that tan $\left\{\frac{\pi}{4} + \frac{1}{2}\, cos^{-1}\frac{a}{b}\right\}$ + tan $\left\{\frac{\pi}{4} - \frac{1}{2}\, cos^{-1}\frac{a}{b}\right\} = \frac{2b}{a}$. **OR**

Q.13) If x = a ($\theta - \sin\theta$) , $y = a\ (1 - \cos\theta), find\ d^2y/dx^2$ at $\theta = \pi/2$.

Q.14) Discuss the continuity of the function, $f(x) = \begin{cases} \frac{x^4 + 2x^3 + x^2}{tan^{-1}x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ at x = 0.

Q.15) Using properties of determinants , prove the following :

$$\begin{vmatrix} 3x & -x+y & -x+z \\ x-y & 3y & z-y \\ x-z & y-z & 3z \end{vmatrix} = 3\,(x+y+z)\,(xy+yz+zx).$$

Q.16) Using vectors find the area of triangle ABC whose vertices are A(1,1,1) , B(1,2,3) and C(2,3,1).

OR

Three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ $satisfy$ the $condition$ $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$ if $|\vec{a}| = 1$, $|\vec{b}| = 4$ and $|\vec{c}| = 2$.

Q.17) Find the distance between the point P(6,5,9) and the plane determined by the points A(3,-1,2), B(5,2,4) and C (-1,-1,6).

OR

Find the image of the point P(5,9,3) in the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$.

Q.18) Find the intervals in which the function $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$ is

i) strictly increasing ii) strictly decreasing.

Q.19) In a group of 20 rich persons, 5 are helpful to poor people. Out of these 20, three persons are selected at random. Write the probability distribution for the selected persons who are helpful to poor people. Also find the mean of the distribution. Explain the values promoted in the question.

Q.20) Solve the following differential equation: $x\frac{dy}{dx} + y = x \log x$; $x \neq 0$.

Q.21) Solve the following differential equation: $x^2\,dy + (xy + y^2)\,dx = 0$, given y = 1 at x = 1.

Q.22) Evaluate $\int_0^{\pi/2} (\sqrt{\tan x} + \sqrt{\cot x})\,dx$.

OR

Evaluate $\int_0^{\pi/4} \log(1 + tanx)\,dx$

SECTION-C

Q.23) Show that the semi vertical angle of right circular cone of maximum volume and of given slant height is $\tan^{-1}(\sqrt{2})$.

OR

Prove that the volume of the largest right circular cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

Q.24) Find the area of that part of the circle $x^2 + y^2 = 16$ which is interior to the parabola $x^2 = 4y$.

Q.25) In answering a question on a MCQ test with 4 choices per question, a student knows the answer, guesses or copies the answer. Let ½ be the probability that he knows the answer, ¼ be the probability that he guesses and ¼ that he copies it . Assuming that a student who copies the answer , will be correct with the probability ¾, what is the probability that the student knows the answer , given that he answered it correctly? What values does a student violate if he uses unfair means?

Q.26) Find the shortest distance between the lines :

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \ and \ \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$$

Q.27) Evaluate $\int_0 \frac{x}{a^2\, cos^2 x + b^2\ sin^2\ x}\ dx.$

OR

Evaluate $\int \frac{(x^2+1)\,(x^2+2)}{(x^2+3)\,(x^2+4)}$ dx.

Q.28) Find the product of the matrices A = $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations : $x - y + z = 4$: $x - 2y - 2z = 9$ and $2x + y + 3z = 1$.

Q.29) A dealer in rural area wishes to purchase a number of sewing machines. He has only Rs. 5760 to invest and has space for at most 20 items. An electronic sewing machine costs Rs. 360 and a manually operated machine costs Rs. 240. He can sell an electronic machine at a profit of Rs. 22 and a manually operated machine at a profit of Rs. 18. Assuming that he can sell all the items that he can buy how he should invest his money in order to maximize his profit . Make it as an LPP and solve it graphically. Keeping the rural background in mind justify the values to be promoted for the selection of the manually operated machines.

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